Flight Testing an Optimal Trajectory for the Saab J35 Draken

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Introduction

THE desire to find the most efficient flight path has a long history. The development of the first high-performance jet aircraft made it crucial to fly a close to optimal path to perform a given mission before the fuel ran out. The earliest methods for finding optimal trajectories were based on calculus of variations. Discretization of the optimality conditions and solution using numerical methods made it possible to solve quite complicated problems; see Bryson and Desai[1] for a discussion on early strategies.

In recent years, the most common approach is to discretize the differential algebraic equations (DAEs) before stating optimality conditions and applying numerical methods. This strategy was pioneered by Hargraves and Paris,[2] who used the NPSOL[3] optimization package. Recent developments in discretization of DAEs[4] and sparse nonlinear optimization methods[5-8] represent the current state of the art in trajectory optimization.

Although many papers appear in the open literature on the development of numerical methods for solving trajectory optimization problems, very few papers are available discussing the difficulties involved in trying to follow a numerically computed trajectory in a real flight test. The purpose of the present paper is to report on a simple flight test that was performed to demonstrate the difficulties that may appear in a flight test.

This paper describes the model and numerical implementation used to calculate optimal trajectories. The presentation of numerical results then follows with the results obtained in the flight test for a particular test case.

Performance Model

The equations of motion for the aircraft assuming a point mass model are given by the system of ordinary differential equations

$$m\dot{V} = T\cos(\alpha + \epsilon) - D - mg\sin\gamma \tag{1}$$

$$mV\dot{\gamma} = T\sin(\alpha + \epsilon) + L - mg\cos\gamma \tag{2}$$

$$\dot{h} = V\sin\gamma \tag{3}$$

$$\dot{x}_E = V\cos\gamma \tag{4}$$

$$\dot{m}_f = -b \tag{5}$$

where m is aircraft mass, T engine thrust, α angle of attack, D drag, g gravity acceleration, γ flight-path angle, L lift, h altitude, x_E distance traveled, and b fuel burn. The mass of the aircraft is split in two parts so that m is the sum of a fixed part and the fuel mass m_f, which is treated as a state variable. The angle ϵ defines the angle of the engine thrust to the fixed body coordinate axis.

The lift and drag are defined through tables of nondimensional aerodynamic coefficients dependent on the state in terms of Mach number, altitude, and position of center of gravity. Each aerodynamic coefficient is modeled as a smooth function using a least-squares fit of B-spline basis functions to the tabular data. The relations between airspeed V, Mach number M, and air density at different altitudes are given through the definition of the standard atmosphere (ISA). The atmospheric model is also defined for arbitrary constant temperature shifts from the ISA defining the relation between pressure altitude and geopotential altitude.

Defining a smooth model of engine thrust and fuel burn as functions of altitude, Mach number, engine control setting, and possibly temperature requires the solution of fairly large least-squares problems. The engine model is given in tabular form listing the thrust and fuel burn for given values of Mach number, pressure altitude, thrust control setting, and ISA condition. Currently, the model assumes that the ISA condition is fixed leaving three independent variables for the thrust and fuel burn functions.

Trajectory Optimization

The equations of motion defined by Eqs. (1–5) can be rewritten in brief form as

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) \tag{6}$$

where the vector of state variables is $\boldsymbol{x} = (V, h, x_E, m_f, \gamma)^T$ and the vector of controls $\boldsymbol{u} = (\alpha, \delta_T)^T$. Additional requirements are implemented as purely algebraic constraints in the form

$$\underline{g} \le g(\boldsymbol{x}, \boldsymbol{u}) \le \bar{g} \tag{7}$$

where $\underline{g}$ and $\bar{g}$ are lower and upper bounds on the algebraic constraints.

By investigating the equations of motion more closely, we can be observe that the distance state variable x_E is not explicitly dependent on the control variables. Further, none of the other state equations depend on x_E making it possible to evaluate the distance at any instance by integrating the time histories of V and γ. Consequently, the distance may be defined as the algebraic function

$$x_E(t_F) = \int_{t=0}^{t_F} V(t)\cos\gamma(t)\,\mathrm{d}t \tag{8}$$

Significant amounts of testing have shown that it appears easier to solve the trajectory optimization problem if the range x_E is implemented as the algebraic constraint (8) as opposed to considering x_E as a state variable in Eq. (6). The number of state variables is reduced by 20% at the cost of an additional algebraic constraint. The drawback is that the algebraic constraint (8) in this form depends on all of the state variables giving a dense row in the constraint Jacobian of the discretized optimization problem.

Currently, the differential and algebraic constraints are defined so that the state variables are $\boldsymbol{x} = (V, h, m_f, \gamma)^T$ keeping the distance (8) together with the algebraic constraints (7). The other algebraic constraints currently implemented concern load factor n_Z, dynamic pressure, lift coefficient, Mach number, and indicated airspeed V_i.

To solve a minimum final time optimization problem, it is necessary to introduce the nondimensional time $\hat{t} = t/t_F$, where t_F denotes the final time. The differential equation defined in nondimensional time becomes

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\hat{t}} = t_F f(\boldsymbol{x}, \boldsymbol{u}) \tag{9}$$

This way, the final time t_F is free to be used as a variable in the discretized optimization problem.

Discretization

The DAEs defined by Eqs. (6) and (7) are discretized using Hermite–Simpson collocation[4] so that state variables $\boldsymbol{x}$ are approximated as piecewise cubic polynomial functions of nondimensional time. The control variables $\boldsymbol{u}$ are approximated as piecewise linear functions of nondimensional time. The differential equations are satisfied in an integral sense on each time step. The algebraic constraints are enforced both in an integral sense on each time step but also at the endpoints of each time step.

The discretization process transforms the differential equations to algebraic equations dependent on the state and control variables at

Received 23 August 1999; revision received 14 September 1999; accepted for publication 14 September 1999.

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selected discretization nodal time points. All of the discretized state and control variables are stored in the finite-dimensional vector $\boldsymbol{y}$ together with the final time variable t_F. The differential and algebraic equations can now be stated as a set of purely algebraic equations dependent on the vector $\boldsymbol{y}$ as

$$\underline{l}_i \leq \boldsymbol{c}_i(\boldsymbol{y}) \leq \bar{\boldsymbol{u}}_i, \qquad i = 1, \ldots, n_c \tag{10}$$

where $\underline{\boldsymbol{l}}$ and $\bar{\boldsymbol{u}}$ define lower and upper bounds on the nonlinear constraints. Equality constraints are simply enforced by setting the lower and upper bounds to be equal.

Optimization

By the defining of an objective function, such as minimum time used for a given mission, it is now possible to formulate the optimization problem as

$$\min_{y} f_0(\boldsymbol{y}) \tag{11}$$

$$\underline{\boldsymbol{l}} \leq \begin{bmatrix} \boldsymbol{c}(\boldsymbol{y}) \\ \boldsymbol{y} \end{bmatrix} \leq \bar{\boldsymbol{u}} \tag{12}$$

where f_0 is the objective function. The vectors containing all of the constraint lower and upper bounds are denoted $\underline{\boldsymbol{l}}$ and $\bar{\boldsymbol{u}}$, respectively. The path constraints on the state and control variables are implemented as the simple bound constraints on the elements of $\boldsymbol{y}$.

Current options for the choice of objective function includes total flight time t_F, fuel mass, or distance. Switching from minimization to maximization is simply done by changing the sign of the objective. There is also a possibility to minimize the deviation from a reference value of Mach number, altitude, or calibrated airspeed. This last feature is useful when analyzing the reference cases defined in the flight manual that usually involves keeping one of these quantities constant for part of the mission.

The optimization problem (11–12) is usually quite large involving many thousands of variables and constraints. Although large scale, the constraint function derivatives are usually quite sparse making efficient numerical solution possible. In this investigation, SNOPT[9] is used to solve the optimization problems inasmuch as it is available through collaboration with the development group at Stanford University and the University of California, San Diego.

Flight Testing: Saab J35 Draken

There is no substitute for flight testing if the objective is to develop a computational model for trajectory optimization that is to be used in practice. However, the number of uncertainties involved in a flight test are numerous, and it can be difficult to establish if there are errors or inaccuracies in the computational model. The major uncertainties involved concern the pilot's ability to follow a complex trajectory and the accuracy of the weather model. Further uncertainties also concern the engine model because there is significant variation between engine individuals, in particular for older aircraft with many hours of use.

The tests presented in this report have been performed by the second fighter wing of the Swedish Air Force base F10 in Ängelholm in southern Sweden.

The venerable Saab J35 Draken is a single engine supersonic interceptor of 1950s design. The planform is quite unusual featuring a double delta wing. Later versions were reportedly Mach 2+ capable in the right circumstances. The aircraft was retired from the Swedish Air Force in December 1998 but is still in service with the Austrian Air Force.

The test presented here was performed during the last days of active service in November 1998. The test aircraft was the J35 Draken, serial number 35601, which was manufactured in 1970 and later upgraded to the J standard. Originally, the test involved two different cases: one minimum time to climb trajectory and one acceleration in level flight. Unfortunately, it was only possible to perform the acceleration case. Because of congested air traffic in the test area, the pilot had to abandon the minimum time to climb in both attempts.

Table 1 Initial and final conditions for the J35 Draken acceleration

Parameter	Initial	Final
M	0.8	1.2
h, km	8	8
γ, rad	0	0

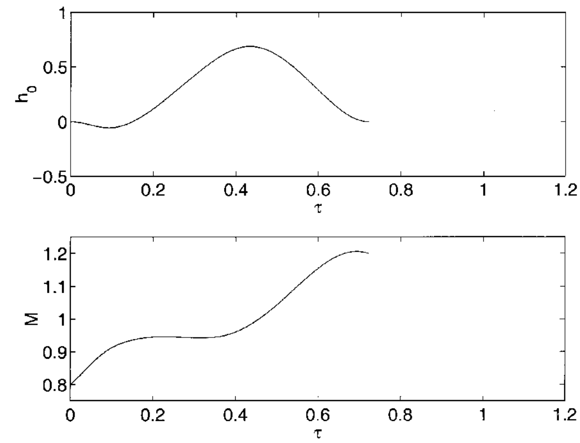

Fig. 1 Optimum acceleration trajectory for the J35 Draken.

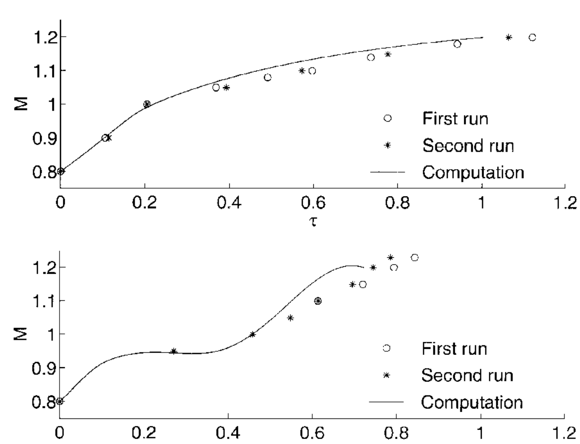

Fig. 2 Flight-test results for the reference (top) and optimum acceleration trajectories.

The acceleration case involves starting at Mach 0.8 at an altitude of 8 km in level flight and accelerating to Mach 1.2 at the same altitude using full thrust with afterburner. The case is specified in Table 1. The acceleration was performed keeping altitude constant at 8 km giving a reference trajectory to be used for comparison with the optimal trajectory.

Solving the optimization problem using the data in Table 1 as initial and final conditions gives the optimal trajectory shown in Fig. 1. Nondimensional altitude h_0 and time τ are used in the graphs because detailed information about performance is still classified. The optimum trajectory shown in Fig. 1 involves a shallow dive quickly reaching the optimal climb speed, which is maintained until the aircraft reaches an altitude of approximately 10 km. The last part of the trajectory involves a dive to supersonic speed reaching the final Mach number in level flight.

The test flight of the reference trajectory was straightforward to perform. Two runs were made, which are shown in the top part of Fig. 2 together with the results obtained using the computational model. The aircraft lacks any means of storing state variables during flight for postprocessing, and so the experimental data shown in the graphs are from notes taken by the pilot during flight.

Testing the optimal trajectory was considered more difficult and so a few practice runs were made in a simulator. The flight test of the optimal trajectory was then performed twice giving the slight differences shown in Fig. 2. However, the difference between the two flight tests is less than the difference between the flight tests and the computation, showing the excellent skills of the pilot in following the prescribed trajectory. Further, it was very encouraging to see that the theoretically predicted time reduction of 28% was so closely reproduced in the flight test, which gave a time reduction of approximately 30%.

Conclusions

It was expected that the ability of the pilot to follow the desired trajectory would be the main uncertainty in the test. However, the present case and further testing currently in progress suggest that it is indeed quite possible to follow even quite complex trajectories with a bit of training in a simulator. However, the longer term goal is to compute the desired trajectory in close to real time and then let an autopilot fly the trajectory significantly reducing pilot workload.

Apart from the uncertainty in engine performance already mentioned, a much more significant uncertainty appears to be the atmospheric model. Standard procedure calls for using the ISA, possibly modified with a constant temperature shift. However, investigating the atmospheric variation as a function of altitude during the tests showed that the conditions are not often well represented by a model involving a constant temperature shift from the ISA. Instead, the model should be based on local information, where temperature and static pressure are given as functions of geopotential altitude. This information is available from the weather service at F10 but is not yet implemented in the computational model.

A significant computational difficulty that often appears when solving trajectory optimization problems is the robustness of the solution method. The optimization problem [Eqs. (11) and (12)] is highly nonlinear, and the algorithm does in practice not always converge to a local optimum. A particular case where the algorithm may fail is when the local quadratic programming model of Eqs. (11) and (12), used to define the next approximation to the solution, does not have a feasible solution even though the original nonlinear problem has a solution. The sequential quadratic programming method[9] used here has the facility to deal with this case but it does not always work. Further difficulties arise when there indeed is no feasible point to the constraints (12). Deciding whether or not there exists a feasible solution is a problem that is at least as difficult as solving the optimization problem itself.

Despite the occasional difficulties described, it is still possible to solve quite general performance optimization problems using the method described. Problems such as a brief acceleration or a climb problem, as well as maximum range problems, can be solved using the same optimization method. Furthermore, the limited flight testing performed also suggest that the results may be quite useful in practice.

Acknowledgments

This project is financially supported by the Swedish Defense Materiel Administration (FMV) Project 61969-LB104302. The author is most grateful for the support provided by the staff of FMV:FlygFT, in particular Martin Näsman. Special thanks are also due to the staff at the Swedish Air Force base F10 in Ängelholm, especially Dan Eriksson and Axel Nilsson, who made the flight tests possible. The author is also most grateful for the support of Philip Gill at the University of California, San Diego, and Walter Murray and Michael Saunders at Stanford University.

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Transonic and Low-Supersonic Aerodynamic Analysis of a Wing with Underpylon/Store

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Introduction

MODERN multipurpose military aircrafts are often equipped with several types of wing-mounted external stores. Generally, external stores are placed on the undersurface and tip of aircraft wing as extra fuel tanks or weapons such as missiles, bombs, rockets, and gun pods. In the transonic and low-supersonic speeds there is strong shock interference between the wing and the pylon/store, which causes a severe vibration of the aircraft wing such as flutter, limit cycle oscillation, and buffet, etc. If the aircraft is under the continuous vibration condition as just mentioned, there are critical structural and fatigue damages of wing structures. So, the correct prediction of instability such as flutter is essential. A typical flutter analysis requires several unsteady aerodynamic computations. In addition, such computations using the method of computational fluid dynamics are quite expensive in the transonic and low-supersonic regime. Therefore, the development of efficient and accurate computational codes for the unsteady aerodynamics is very important for the flutter analysis.

The detailed wind-tunnel experiments for the F-5 fighter wing had been conducted at the National Aerospace Laboratory of the Netherlands (NLR) under the sponsorship of the U.S. Air Force.[1] It could be shown from NLR's experiments that there was clearly a strong influence of the underpylon/store on the steady and unsteady transonic aerodynamics. From the 1970s there were great efforts to develop efficient unsteady aerodynamic codes based on the three-dimensional transonic small-disturbance (TSD) theory. As the results of previous researches, the famous and very efficient codes such as ATRAN3S[2,3] and CAP-TSD[4,5] have been developed and verified for several application cases.

Received 10 August 1999; revision received 7 September 1999; accepted for publication 7 September 1999.

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